Economics 501B Final Exam
University of Arizona
Fall 2011

1. Let X be a set of possible *social outcomes* — for example, X could be $\mathbb{R}^{nl}_+$ and the social outcomes x in $\mathbb{R}^{nl}_+$ could be allocations of l goods to n consumers. Suppose each individual $i = 1, \ldots, n$ has a preference over the elements of X and each preference is represented by a utility function $u^i : X \to \mathbb{R}$. Provide proofs of the two propositions in (a) and (b):

(a) **Proposition:** If the outcome $\hat{x}$ is Pareto efficient, then $\hat{x}$ is a solution of the following maximization problem:

$$\max_{x \in X} u^1(x) \text{ subject to } u^i(x) \geqq u^i(\hat{x}), \quad i = 2, ..., n.$$

(b) **Proposition:** If the outcome $\hat{x}$ is a solution of the problem

$$\max_{x \in X} W(x) := \sum_{i=1}^{n} \alpha_i u^i(x)$$

for some numbers $\alpha_1, ..., \alpha_n > 0$, then $\hat{x}$ is Pareto efficient.

For parts (c) and (d) assume that X is a set of allocations of two goods to two people:

$$X = \{ \mathbf{x} \in \mathbb{R}^4_+ \mid x_{1k} + x_{2k} \leqq \mathring{x}_k, \ k = 1, 2 \}.$$

(c) Provide a counterexample to show that the converse of the proposition in (a) is not true — *i.e.*, that a solution of the maximization problem in (a) may not be a Pareto allocation.

(d) Provide a counterexample to show that the converse of the proposition in (b) is not true — *i.e.*, provide utility functions and a Pareto allocation which is not a solution of the maximization problem in (b) for any α_1 and α_2.

2. The only good in the economy is oranges, and Amy and Beth are the only two persons. An orange tree will yield 30 oranges today and 30 oranges again tomorrow, and today's oranges cannot be stored until tomorrow: they will spoil, whether they are picked or they are left on the tree. Amy owns one tree (and all the oranges it will produce) and Beth owns two trees (and their oranges). The weather may be very hot tomorrow (we'll call that State H), or it may not (we'll call that State L). Neither Amy nor Beth knows for sure today whether it will be hot tomorrow or not, and in fact Beth's preference for oranges is unaffected by tomorrow's weather: her preference can be represented by the utility function

$$u^B(x_{B0}, x_{BH}, x_{BL}) = x_{B0} + 15 \ln x_{BH} + 15 \ln x_{BL} \ ,$$

where x_{B0} denotes her consumption of oranges today and x_{BH} and x_{BL} her state-contingent consumption tomorrow. Amy, however, would be willing (today) to give up some anticipated consumption of oranges in cool weather tomorrow if that would enable her to consume more oranges should it turn out to be hot tomorrow: her preference is described by the utility function

$$u^A(x_{A0}, x_{AH}, x_{AL}) = x_{A0} + 30 \ln x_{AH} + 15 \ln x_{AL}.$$

Tomorrow's weather will have no effect on the number of oranges the trees will bear.

(a) Determine all the Pareto efficient allocations of today's and tomorrow's oranges.

(b) Determine the Arrow-Debreu complete-contingent-claims market prices, assuming the price of an orange today is one dollar.

(c) Suppose there are *not* complete markets, but instead only a market in which one can borrow and lend (with repayment at a rate that does not depend upon tomorrow's weather). Determine the equilibrium allocation and rate of interest. How much will each woman borrow or lend?

(d) If the allocation in (c) is Pareto efficient, verify it. If it is not, find a Pareto improvement.

(e) Now suppose that, in addition to the borrowing-and-lending market in (d), there is also a security available that will yield two oranges tomorrow if the weather is hot, but only one orange if the weather is cool. Use the Arrow-Debreu prices to determine the interest rate and the price of the security in equilibrium. What will be the equilibrium consumption of oranges by Amy and Beth? What will each woman's portfolio holding be in equilibrium?

3. Arnie, Ben, and Chris own a golf course. The quality of the greens depends on the number of dollars they spend on maintenance. Let x denote the quality of the greens and let y_i denote the number of dollars player i spends on other goods. Player i's preference is described by the utility function $u_i(x, y_i)$. The cost of maintaining the greens at level x is $C(x)$ dollars.

(a) Derive the Samuelson marginal condition that characterizes the interior Pareto allocations.

From now on assume that $C(x) = 3x$ and that the utility functions are $u_i(x, y_i) = y_i + \alpha_i \ln x$, where $\alpha_1 = 1$, $\alpha_2 = 2$, and $\alpha_3 = 3$, and assume that each player's wealth is \$100.

(b) Determine the set of Pareto allocations.

(c) Determine the Lindahl prices and the Lindahl allocation.

(d) Suppose the level of maintenance (i.e., x, the quality of the greens) is determined via voluntary contributions. Let t_i denote the amount each player contributes. Determine the Nash equilibrium level x of maintenance and the associated contribution by each player.

(e) Now suppose that the players have agreed to use the following method each week to determine that week's level of maintenance x and the payments t_1, t_2, and t_3 each player will make: Each will place a *request* r_i with the maintenance company; the company is then authorized to maintain the greens at level $x = r_1 + r_2 + r_3$ and to charge the players the amounts

$$t_1 = (1 + r_2 - r_3)x \qquad t_2 = (1 + r_3 - r_1)x \qquad t_3 = (1 + r_1 - r_2)x.$$

Note that $t_1 + t_2 + t_3$ will always be equal to $3x$. Verify that the Nash equilibrium (r_1, r_2, r_3) yields the Lindahl allocation.